Financial Mathematics in Economics

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About This Book:

Financial Mathematics in Economics introduces readers to basic concepts in finance that require knowledge of mathematics and economics. The areas to be covered are simple interest and simple discount; compound interest and compound discount; bank discount, trade discount, and cash discount; annuity; and depreciation. Each chapter begins with a learning outcomes section and ends with a review exercise as part of a learning reflection for readers. Finally, reader outcomes include financial applications in work and everyday life.

The book is organised into six chapters and then divided into subchapters by concept or topic. Interestingly, it applies interactive learning aspects to help readers have fun and learn quickly. Manual calculation and derivation of formulas are explained in detail and consistent terminology and notation are adapted in the text to keep continuous understanding throughout the text. Manual calculations are essential for readers to understand the sources of financial change and ultimately achieve money literacy and economic stability.

For the Reader

Each chapter in this book begins with learning outcomes and concludes with a concept review that highlights the key topics in the chapter and review exercises. As an interactive book, readers need to interact with the text, where some chapters have interactive videos with supplement assessments to enhance the knowledge sharing outcomes.

Feedbacks

To provide feedback on this book, please contact *sarimah.surianshah@ums.edu.my*

PART I

CHAPTER 1: INTRODUCTION TO FINANCIAL MATHEMATICS IN ECONOMICS

Learning Outcomes

At the end of this chapter, you should able to:

- Describe the relevance of financial mathematics in economics.
- Explain economic gains for individuals and firms.
- Determine where financial mathematics fits.

1.1 Introduction

Figure 1
Adopted from “Money saving growth image” by nattanan is licensed under CC BY 3.0

Take a look at the image above. This image illustrates the benefits of saving and the importance to stablise it in life. How to maintain the stabilisation? In this course, we will learn a few ways to make a financial decision by considering the economic aspects such as inflation and any uncertainty. However, note that every decision has its opportunity cost or missed opportunity. But it can be minimised through better financial decision-making.

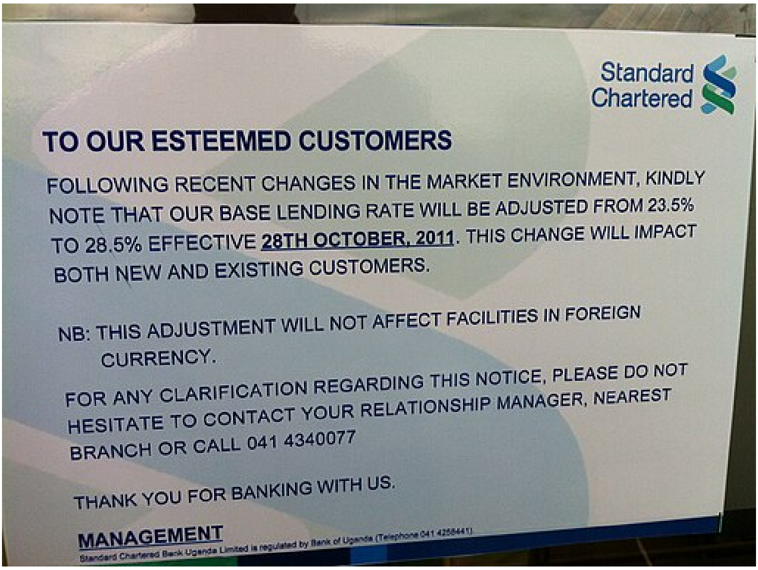

Figure 2

Adopted from ["Picture att note in standard chartered bank"](#) by Mobergefinance is licensed under [CC BY 3.0](#)

Having a huge amount of savings doesn't mean having financial freedom. A good financial management is needed. Figure 2 shows a choice for an individual to apply for a loan from a company or other financial institution. What elements should be considered before making the choice? Is it more profitable to choose Figure 2's option or institutions with better bank discounts? Is the term of payment offered fruitful?

In this book, and each chapter, you will begin to explore the definition of the terminology of the title, the components required for the terminology, the calculation, and finally the application in finance and economics aspects.

1.2 Financial Mathematics: Where does it apply?

Financial mathematics helps in evaluating any financial decision that may be related to economic conditions. For example, borrowers need to consider the interest rate given when making a loan. Meanwhile, the interest rate is closely related to the inflation rates. The inflation rate increases if the currency rate for the home country depreciates caused by economic uncertainties such as political instability in the country. Finally, the depreciation causes economic gains for individuals or firms to decrease.

Financial mathematics also is needed for many kinds of decision-making. For instance, buying a car. Given two alternatives, $18,000 now, or $600 per month for 3 years. Which is better? It depends! The issue is how much is money now worth compared to money in the future. It leads to the key concept of the time value of money.

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=31#h5p-1

Finally, financial mathematics applies in your personal life. In general, approaches to problem solving are: i) understand the problem; ii) collect all relevant data or information; iii) define the feasible alternatives; iv) evaluate each alternative; v) select the "best" alternative; vi) implement and monitor the decision. The major role of financial mathematics is applied to approach vi which is to evaluate different alternatives. It involves techniques of formulating, estimating, and evaluating economic outcomes where choices or alternatives are available. Using specific mathematical relationships, individuals will compare the cash flows of the different alternatives. As a resolver, a sensitivity analysis can be performed to explore how the decision changes as the estimates change.

1.3 Review Exercises

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=33#h5p-2

1.4 Highlights

- Financial mathematics is needed for many aspects of economics.
- The role of financial mathematics in economics can be reviewed from established facts such as the interest rate.
- Financial mathematics helps make cash flow comparisons.
- Financial mathematics is just a set of tools to help in decision making, but the best decision is up to your expected opportunity cost.

1.5 Key Terms

Depreciation

Economic gains

Inflation

Interest rate

Opportunity cost

Saving

Time value of money

PART II

CHAPTER 2: SIMPLE INTEREST AND SIMPLE DISCOUNT

Learning Outcomes

At the end of this chapter, you should able to:

- Define simple interest and simple discounts.
- Evaluate the present and future value of borrowed or invested money.
- Calculate simple interest and simple discounts.

2.1 Introduction

Interest is the compensation one gets for lending a certain asset. For example, suppose that you put some money in a bank account for a year. Then, the bank can do whatever it wants with the money for a year. To reward you for that, it pays you some interest. The asset being lent out is called the capital.

In general, both the capital and interest are expressed in money. However, that is not necessary. For instance, a farmer may lend his tractor to a neighbour, and get a part of the grain harvested in return. In this course, the capital is always expressed in money, or called capital.

2.2 Simple Interest

Interest is the reward for lending capital to somebody for a period of time. There are various methods for computing the interest. For simple interest, is it the amount of interest in the product of three quantities which are: i) the rate of interest; ii) the principal; iii) the time period.

From a lender's perspective, when money is borrowed for a loan, interest accumulates as a reward for the lenders. While, from the borrower's perspective, accumulated interest is a charge to the borrower for the financial transaction to take place. The amount of interest incurred by the borrower depends on the amount of money borrowed or invested, the principal, the interest rate, and time. The simple interest, I, accumulated on the principal, P, over an interval of t years at an annual interest rate of r, can be written as follows:

(1) $$I = P \times r \times t$$

Where the rate of interest must be expressed as a decimal for calculations, the interest rate is expressed in year or annually, and the interest rate is a flat rate where there are no changes happens for the interest rate during the tenure period. To note that, p.a. represents per annum or annual interest rate.

2.2.1 Future Value and Present Value

The total amount of money that must be repaid on a loan or the total value of an investment can be called the future value, S. The future value can be calculated using $S = P + I$, where P is the principal or money borrowed for a loan or invested, and I is the interest accumulated. The future value also can be calculated using the following formula, given information on the accumulated amount of the principal and interest after t years:

(2) $$S = P + I = P + (Prt) = P(1 + rt)$$

The principal is also called the present value of the discounted value of S. In Equation (2), $(1 + rt)$ is called the simple interest factor and $(1 + rt)^{-1}$ is called the present value discount factor at simple interest. The time, t, must be in years. When the time is given in months, then

(3) $$t = \frac{numberofmonths}{12}$$

When the time is given in weeks, then divide the weeks with 52, when the time is given in days, then divide the days with 365 days.

The present value (or discounted value) of S was calculated by using the present value factor at simple interest. The present value can be written as Equation (4) :

(4) $$P = \frac{S}{1 + rt}$$

Example 2.1

Find the simple interest on a RM1,000 investment made for 3 years at an interest rate of 5% per year. What is the accumulated amount?

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from this version of the text. You can view it online here:
https://openbook.ums.edu.my/financialmathematicsineconomics/?p=44#h5p-3

Example 2.2

A student borrows RM600 to buy a camera. The loan is over two years, and the simple interest rate is 6% per annum. How much will his/her monthly repayments be?

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=44#h5p-4

Example 2.3

Find the present value of RM800 at a simple interest rate of 10% p.a. for 8 months.

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=44#h5p-5

2.2.2 The Time between Dates

Figure 2
"Calendar" by Nick Youngson is licensed under CC BY-SA 3.0

In any financial transaction, loan terms are an important aspect to be considered before signing off. These include the loan's repayment period. The loan repayment period or time can be calculated using two ways which are i) the exact time, and ii) the approximate time.

Exact time is found as the exact number of days including all days except the first. The exact time can be refereed using the table of the number of each year of the day (see Table 1). It is obtained as the difference between serial numbers of the given dates. For example, to find the exact time from April 18 to November 3 of the same year, see Figure 1. May 18 is the 108th day of the year and November 3 is the 307th day of the year. The exact time is 307 − 108 = 199 days. Alternatively, use the Microsoft Excel.

Table 1

Date	Month	Day
November 3	10	33
April 18	4	18
Difference	6	15

Whereas, the approximate time is calculated by assuming that each month has 30 days. Using the same example as above, see Table 1, for the solution.

Where we have borrowed 30 days from the 11th month. The approximate time is 6 months and 15 days, or (6 x 30 days) + 15 days = 195 days.

2.3 Simple Interest using Sequences

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=46#h5p-6

This is a video about simple interest using sequences. Please answer all questions in the video.

2.4 Simple Discount

2.4.1 Simple Discount at an Interest Rate

In discounting at simple interest, the difference $D = S - P$ is called the simple discount (on S) at an interest rate (r). We may interpret D either as the interest I on P which when added to P gives S, or as the true discount on S which when substracted from S gives P.

2.4.2 Simple Discount at a Discount Rate

The discount rate d for a year is the ratio of the discount D for the year to the amount S on which the discount is given. The simple discount D on an amount S, also called bank discount, for t years at the discount rate d, which can be calculated as follows:

(1) $D = S \times d \times t$

And the discounted value, or proceeds, P of S is given by

(2) $P = S - D = S - Sdt = S(1 - dt)$

The charge for some short-term loans may be based on the final amount rather than on the present value. The lender calculates the bank discount D on the final amount S that must be paid on the due date and deducts it from S; the borrower receives the proceeds P. For this reason, bank discount is sometimes called interest in advance. The following equation calculate the maturity value of a loan for specified proceeds/ principals/ present values of S.

(3) $$S = \frac{P}{1 - dt} = P(1 - dt)^{-1}$$

Example 2.4

Find the present value of 12% simple discount of $1000 due in 5 months. What is the simple discount?

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=48#h5p-7

Example 2.5

Calculate the present value of RM1000 due in 1 year at a simple discount rate of 10% p.a.

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online here:
https://openbook.ums.edu.my/financialmathematicsineconomics/?p=48#h5p-8

2.5 Review Exercises

1. At what rate of simple interest will $1200 accumulate interest of $72 in 6 months?
2. Eighty days after borrowing money, a person pays back exactly $850. How much was borrowed if the $850 payment includes principal and simple interest at 9 ¾ %?
3. Find the present value at 12% simple discount of $1000 due in 5 months. What is the simple discount?
4. A bank charges 12% simple interest in advance. A borrower needs $2000 cash, to be repaid with interest in 9 months. What size loan should he ask for, and how much interest will he pay?

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=50#h5p-9

2.6 Highlights

- Interest is the fee charged to the borrower.
- The sum of the principal and interest due is called the amount or accumulated value.
- The difference between accumulated value, S, and principal, P is called the simple discount.
- A bank discount is also called interest in advance.

2.7 Key Terms

Discount rate
Interest
Interest rate
Present value
Principal
Simple interest
Time period

PART III

CHAPTER 3: COMPOUND INTEREST AND COMPOUND DISCOUNT

Learning Outcomes

At the end of this chapter, you should able to:

- Define compound interest and compound discounts.
- Calculate compound interest and compound discounts.
- Compare nominal and effective interest rates.

An interest paid only on the principal is called simple interest. When the interest of each period is added to the principal in computing the interest for the next period, it is called compound interest.

A simple discount on an amount of transaction for t years at the discount rate of d is called a simple discount at a discount rate. When there is a compound discount at a discount rate, it is called a compound discount at a discount rate.

Both interest and discount rates may be converted into principal annually, semiannually, quarterly, monthly, weekly, daily, or continuously. The number of times or time period of interest is converted in a year, or compounded per year, is called the frequency of conversion. The nominal rate is the common unit used in the calculation of interest or discount rate.

3.1 Compound Interest

An interest paid only on the principal is called simple interest. When the interest of each period is added to the principal in computing the interest for the next period, it is called compound interest.

Interest amount computed at the end of a period is added to a single principal sum. That is, at the end of each interest period the interest earned for that period is added to the principal so that the interest also earns interest over the next interest period. In the same way, interest due on a debt at the end of a period is subject to interest in the next period.

Suppose that P is deposited at a rate of interest r per year. The amount on deposit at the end of the first year is found by the simple interest formula with t = 1;

(1) $\mathrm{S = P(1 + r\ .1)}$

If the deposit earns compound interest, the interest earned during the second year is paid on the total amount on the deposit at the end of the first year. Using the formula $S = P(1 + rt)$ again, with P replaced by $P(1 + r)$ and t = 1, gives the total amount on deposit at the end of the second year;

(2) $\mathrm{S = [P(1 + r\ )]\ (1 + r\ .1)}$

In the same way, the total amount on deposit at the end of the third year is;

(3) $\mathrm{S = P(1{+}r)^3}$

Generalizing, in t years the total amount on deposit is;

(4) $\mathrm{S = P(1{+}r)}^t$

called the compound amount.

3.1.1 The Formula

The compound amount can be calculated using the following formula:

Compound amount:

(5) $$S = P(1 + i)^n$$

where i = r/m and n = mt ;

S = future (maturity) value, or the compound amount of P, or the accumulated value of P

P = original principal, or the present value of P, or the discounted value of S

i = interest rate per period

r = annual interest rate

m = number of compounding periods per year

n = number of compounding periods

t = number of years

Example 3.1

Consider a sum of RM8,200 is deposited into a time deposit account today that pays 5% p.a. How much will it be in the next 5 years if compounded quarterly?

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=58#h5p-10

Example 3.2

Ali invests RM5000 at 6.2% p.a. with interest compounded monthly. What would his investment be worth after five years? What amount of interest has been earned during the five years?

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=58#h5p-11

Example 3.3

How much money will be required on 31 December 2003 to repay a loan of RM2000 made on 31 December 2000 if i=12% compounded quarterly?

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=58#h5p-12

Example 3.4

What amount of money invested today will grow to RM1000 at the end of 5 years if i=18% compounded quarterly?

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=58#h5p-13

3.2 Compound Interest using Sequences

Let P is the principal sum and i the interest rate per annum, mathematically, we can calculate the total balance at the ends of successive years for n years as:

At the end of the 1^{st} year:

The interest due = Pi, then the future value = P + Pi = P(1+i)

At the end of the 2^{nd} year:

The interest due = [P(1+i)]i, then the future value = P (1+i) + [P(1+i)]i

= P(1+i) (1+i)

= $P(1+i)^2$

At the end of the 3^{rd} year:

The interest due = $[P(1+i)^2]i$, then the future value = $P(1+i)^2 + [P(1+i)^2]i$

= $P(1+i)^2(1+i)$

= $P(1+i)^3$

and so on.

Here, we can see that the total balance or the future balance value at the end of n years forms a geometric progression with first term A and common ratio (1+r). Thus, to obtain the total balance at the end of n years, we can use the above formula for the nth term of a geometric progression with n-1 replaced by n as follows:

$S = P(1+i)^n$

Here, the total balance at the beginning of the nth year is given by:

$S = P(1+i)^{n-1}$

However, to note that the above calculation is express as rates per annum or called as nominal rates. In some cases, the interest period need not be a year and for example, interest is "payable half-yearly' or 'compounded half-yearly', interest period is payable half a year. Then, the interest rate need to be divided by two to give the effective rate per half year. The number of times you pay interest or compounding on your principal is important to calculate the total balance or future value.

3.3 Compound Discount

3.3.1 Compound Discount at a Discount Rate

Let d^m be the nominal rate of discount compounded m times per year.

Then the discount rate per conversion period is d^m/m and the discounted value *P* of a future amount *S* due in *n* periods is

(1) $$P = S(1 - \frac{d^m}{m})^n$$

Thus, the accumulated value S is:

(2) $$S = P(1 - \frac{d^m}{m})^{-n}$$

Example 3.5

Find the discounted value of RM1000 due in 2 years at d =12% compounded monthly.

An interactive H5P element has been excluded from this version of the text. You can view it online here:

https://openbook.ums.edu.my/financialmathematicsineconomics/?p=62#h5p-14

Example 3.6

Find the discounted value of RM1000 due in 2 years at d=7% compounded daily.

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://openbook.ums.edu.my/financialmathematicsineconomics/?p=62#h5p-15

3.4 Effective Interest Rate

Effective interest rates can be used to compare investment opportunities. The effective rate of interest is the equivalent rate of simple interest earned over one year for an interest rate that is compounded twice or more over the year. The annual simple interest rate will be greater than the annual compounding interest rate to earn the same amount of interest.

The annual interest rate, r, for compounding calculations, is often called the nominal rate.

Illustration: If RM1 is deposited at 4% compounded quarterly, the compound amount is RM1.0406, an increase of 4.06% over the original RM1. Here, the actual increase of 4.06% in the money is somewhat higher than the stated increase of 4%. To differentiate between these two numbers, 4% is called the nominal or stated rate of interest, r while 4.06% is called the effective rate, r_E .

The effective rate corresponding to a stated rate of interest r compounded m times per year can be calculated as follows:

(1) $$r_E = (1 + \frac{r}{m})^m - 1$$

Example 3.7

Betty needs to borrow money. Bank A charges 8% interest compounded semiannually. Bank B charges 7.9% interest compounded monthly. At which bank will she pay the lesser amount of interest?

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=64#h5p-16

3.5 Review Exercises

Select TRUE or FALSE for each of the following statements.

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3.6 Highlights

- If the interest is added to the investment at the end of each period, and thereafter also earns interest, the investment earns compound interest.
- As the discount rate is compounded m times per year, compound discount at a discount rate should be calculated.
- For a given nominal rate compounded m times per year, the corresponding effective rate of interest is defined as the rate that will produce the same amount of interest per year.

3.7 Key Terms

Compound interest

Discount rate

Interest

Interest rate

Principal

Time period

PART IV

CHAPTER 4: BANK DISCOUNT, TRADE DISCOUNT, AND CASH DISCOUNT

Learning Outcomes

At the end of this chapter, you should able to:

- Define bank discount, trade discount, and cash discount.
- Calculate bank discount, trade discount, and cash discount.
- Define the elements of negotiable instruments.

A bank discount is applied at the time that the note or loan is extended, and is automatically deducted from the loan amount that is used to calculate the schedule of payments on the loan.

A trade discount is a discount that is cut from the retail or published price of an item. It helps to increase the demand for the goods. Turning into a cash discount, it is offered to customers from the selling price aiming to encourage an early payment before the expiration of the credit period.

After completing the negotiable activities using either one of the above, a negotiable instrument is used to guarantee the payment.

4.1 Bank Discount

A bank discount is a bank charge that is made for payment of a note at some point prior to maturation. It is applied at the time that the note/loan is extended, and is automatically deducted from the loan amount that is used to calculate the schedule of payments on the loan. This means that the receiver of the loan simply repays the face value of the loan with little/no interest.

To receive a bank discount, an individual or a firm should possess a solid record of previous financing with the institution. If the past loan history shows no late payments and no complications with the loans, then the chances of receiving a bank discount are greatly improved.

The level of bank credit is also a factor. The eligibility for receiving a bank discount is impacted by the current assets and liabilities of the borrower. If there is a high credit rating, then the chances of getting a bank discount are enhanced. These types of customers are considered as good credit risks with the bank can afford to extend a bank discount with the expectation of doing more business with the customers in the future.

4.1.1 The Formula

In section 2.4.2, we deal with a simple discount at a discount rate. Similarly, we can calculate the bank discount (D) as follows;

D = Sdt

where S is the maturity value, d is the discount rate, and t is time.

The money received for the discounted note is called the proceeds. The proceeds (P) are obtained by deducting the bank discount (D) from the maturity value (S) of the note:

P = S-D

From both formulas, we can calculate the proceeds as follows:

$P = S - D = S - Sdt = S(1 - dt)$

4.2 Trade Discount

A trade discount is a discount that is cut from the retail or published price of an item. It helps to increase the demand for the goods. Customers who purchase goods in larger quantities including wholesalers, retailers, and industrial users tend to receive trade discounts. In general, the larger the purchase, the bigger the trade discount.

In general, there are two types of merchants or demanders of goods in the market, the wholesaler and the retailer. Wholesalers are those who usually purchase goods from manufacturers and producers and sell them to retailers and large consumers. While, retailer merchants are those who purchase goods from several sources, including wholesalers, producers, manufacturers, and agent middlemen such as brokers and manufacturers' agents. They mainly sell goods to the ultimate consumers.

Wholesalers normally tend to buy in larger amounts than the retailers, and the retailers tend to buy in larger amounts than the consumers. Manufacturers, wholesalers, or other types of sellers frequently grant substantial reductions from the list price quoted in their catalogs to allow for price differentials. Such reductions based on the list price are called trade discounts. Merchants do not need to reprint the catalog when offering trade discounts, and the trade discount is not fixed as it depends on the market situations that control the price fluctuation.

Besides being offered to customers who purchase in bulk, trade discounts also are offered based on customer location, order size, and the customer credit rating. Customers may have the opportunity to receive multiple trade discounts known as chain discounts. When a chain discount or series discount is applied, each subsequent discount is deducted from the balance of the previous discount.

Figure 1: Example of trade discount offered. "Borders Books Ad 2 | The second going out of business notice… |" by Mark Mathosian is licensed under CC BY 4.0

Among economic reasons, merchants offer trade discounts are:

1. To create healthy competition between firms
2. To finish old/ unused stocks.
3. To have a quick sale of the stocks before the expiry date.
4. To encourage sales in terms of bulk or large amounts.

4.2.1 The Formula

The trade discount for a single discount can be computed using the following formula:

Trade discount = List price x Discount rate

Where the selling price is the difference between the list price and the trade discount given, it can be written as:

Selling price = List price – Trade discount

Whereas, the trade discount for chain discount, assumed for three discounts, can be computed as:

Trade discount = List price x [1- ((1-First discount rate)(1-Second discount rate)(1-Third discount rate))]

With the following selling price:

Selling price = List price – Trade discount

Example 4.1

The price of a ring listed in a catalog is RM600. The price is subject to a discount of 20%. What are the trade discount and the selling price?

Example 4.2

A TV cost RM2000. A single trade discount is offered at 40%. Find the discount value and the price of the TV sold at discount price.

Example 4.3

Calculate the equivalent trade discount and the net price of the computer sold at RM5000 with series discounts of 30/20/10.

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https://openbook.ums.edu.my/financialmathematicsineconomics/?p=77#h5p-22

4.3 Cash Discount

There are considerable variations in the methods of payment among different types of businesses. Some firms require immediate payment, while others allow their customers to pay bills within a specified period known as the credit period. If used properly, cash discounts also known as purchase discounts can improve the days-sales-outstanding aspect of a business's cash conversion cycle.

Many companies offer their customers a discount from the selling price called a cash discount. A cash discount is also called a sales discount by the seller, and a purchase discount by the buyer. Such a discount is to induce an early payment before the expiration of the credit period.

Among reasons to offer cash discount are:

1. To collect receivables quickly. Providing a small cash discount would be beneficial for the seller as it would allow him to have access to the cash sooner. The sooner a seller receives the cash, the earlier he can put the money back into the business to buy more supplies and/or grow the company further.

2. Faster collections reduce debts.
3. Sellers can pay their own bills and hence reduce interest costs.
4. Competitiveness.
5. To finish old stocks/unsalable goods/poor conditioned stocks.
6. To encourage goods sold in bulk/large quantities.

In general, a cash discount term is written as "2/10, n/30". It means if a payment is made within 10 days from the date of the invoice, a 2% cash discount is allowed, although the debtor is permitted a period of 30 days to pay the bill. However, if the bill is paid after the end of 10 days but on or before the end of the 30-day period, the net amount of the invoice must be paid. After 30 days, the bill will be considered overdue and may be subjected to an interest charge.

However, note that freight costs are not subject to cash discounts and need to be deducted from the invoice.

4.3.1 The Formula

The amount of cash discount is given as:

Cash Discount = Invoice amount (or selling price) x Cash discount rate (%)

Subject to a few conditions, such as:

Normal date: 2/10, n/30.

End of the Month (E.O.M.): 2/10 E.O.M.

Received of Goods (R.O.G.): 2/10 R.O.G.

E.O.M means the discount period starts from the end of the month instead of the invoice date.

R.O.G. means the discount period starts from the date of receipt of the goods.

Example 4.4

RM30000 invoice dated March 10 with terms of 1/10, N/30.

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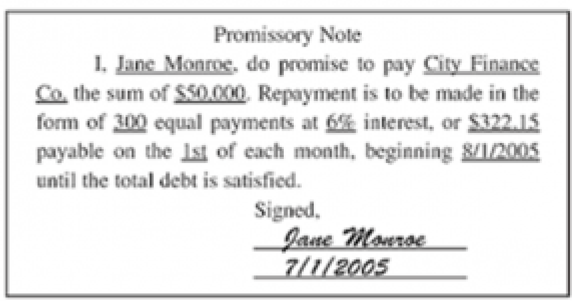
4.4 Negotiable Instruments

A negotiable instrument is a document guaranteeing the payment of a specific amount of money, either on demand or at a set time, with the payer named on the document. It is a document contemplated by or consisting of a contract, which promises the payment of money without condition, which may be paid either on demand or at a future date. Examples: promissory notes, bills of exchange, banknotes, and cheques.



Promissory Note

I, Jane Monroe, do promise to pay City Finance Co. the sum of $50,000. Repayment is to be made in the form of 300 equal payments at 6% interest, or $322.15 payable on the 1st of each month, beginning 8/1/2005 until the total debt is satisfied.

Signed,

Jane Monroe

7/1/2005

FIG. 152. PROMISSORY NOTE

Figure 2: Example of a negotiable instrument

From the figure above, the total of principal and interest is called the maturity value of the note. Given the term of the note is in months, ordinary simple interest is used to calculate the maturity value. An important feature of a promissory note is that it is negotiable. Negotiable, that is, it can be transferred to another payee by the endorsement of the present payee.

4.5 Review Exercises

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://openbook.ums.edu.my/financialmathematicsineconomics/?p=84#h5p-24

4.6 Highlights

- Bank discount is a bank charge that is made for payment of a note at some point prior to maturation.
- Trade discount is a discount that is cut from the retail or published price of an item.
- A cash discount is also called a sales discount by the seller, and a purchase discount by the buyer. It aims to induce early payment.
- A negotiable instrument is a document guaranteeing the payment of a specific amount of money, either on demand or at a set time, with the payer named on the document.

4.7 Key Terms

Bank discount

Cash discount

Negotiable instrument

Trade discount

PART V

CHAPTER 5: ANNUITY

Learning Outcomes

At the end of this chapter, you should able to:

- Define the two types of annuity.
- Differentiate between ordinary annuity and annuity due.
- Understand the calculation between the present and future value of ordinary annuity and annuity due.

An annuity is a sequence of equal payments made at equal periods. Example: Insurance premium, mortgage payments, interest payments on bonds, payments of rent, payments on hire purchases, dividends, etc.

There are two types of annuity which are ordinary annuity/ annuity-immediate and annuity due. If the payments are made at the end of the time period, and if the frequency of payments is the same as the frequency of compounding, the annuity is called an ordinary annuity/ annuity-immediate. While annuity due is an annuity whose periodic payments are paid at the beginning of each payment period.

5.1 Definitions

An annuity is a sequence of equal payments made at equal periods of time. Insurance premiums, mortgage payments, interest payments on bonds, payments of rent, payments on hire purchases, and dividends are a few examples of annuities.

When the payments are made at the end of the time period, the annuity is called an ordinary annuity. Examples include loan repayments and interest payments on bonds. When the payments are paid at the beginning of each payment period, the annuity is called an annuity due. Examples include insurance premiums.

The future value (or accumulated value) of an annuity is the total value of the set of payments at the end of the term. Similarly, the present value (or discounted value) of an annuity is the equivalent value of the set of payments due at the beginning of the term.

5.2 Future Value of An Ordinary Annuity

The future value of an ordinary annuity is defined as the amount due at the end of the term that is equivalent to the sum of the future value of all the payments comprising the annuity, with the date of the last payment as the focal date. An ordinary annuity is shown on the time diagram below.

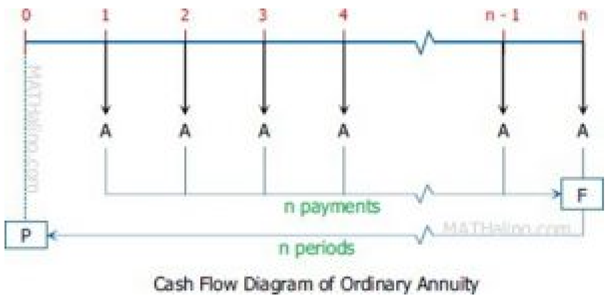


Figure 1: Illustration of an ordinary annuity timeline

To calculate the future value of this annuity, we need to accumulate each payment at the end of the term. This may be expressed as follows:

$= (1 + i)^{n-1} + (1 + i)^{n-2} + \ldots + (1 + i)^{2} + (1 + i)^{1} + 1$

Written this in the other way around, we have:

$= 1 + (1 + i)^{1} + (1 + i)^{2} + \ldots + (1 + i)^{n-2} + (1 + i)^{n-1}$

This expression is a geometric progression with n terms, where the first term is 1 and with a common factor of $(1 + i)$. Hence, using the formula of the sum of a geometric progression, we can generate the future value as follows:

$=\frac{a(r^n-1)}{r-1}$

$= \frac{1[(1+i)^n - 1]}{(1+i) - 1}$

$= \frac{(1+i)^n - 1}{i}$

The future value at i of RM1 paid at each period for n periods is often written as:

$s_{\bar{n}|i}$, read "s angle n at i" where;

$$s_{\bar{n}|i} = \frac{(1+i)^n - 1}{i}$$

The future value of an ordinary annuity of n payments of RMR each, multiply $s_{\bar{n}|i}$ by R and is written as:

Future value = $Rs_{\bar{n}|i} = R\frac{(1+i)^n - 1}{i}$

Example 5.1

A worker is saving RM1000 each year and depositing it into Bank A. How much money will she have at the end of 40 years for her retirement if the interest rate is 9% p.a.?

Example 5.2

Find the future value of an annuity of RM1500 payable at the end of each month at i_{12} = 8% for 10 years.

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5.3 Present Value of An Ordinary Annuity

The present value (or discounted value) of an annuity is the equivalent value of the set of payments due at the beginning of the term which is one period before the first payment and is equivalent to the sum of the present values of all the payments comprising the annuity.

The relationship between the present value and future value can be written as:

Present value, $a_{\bar{n}|i}$ = Future value × $(1 + i)^{-n}$

Substitute the formula of future value in section 5.2 we have:

$$a_{\bar{n}|i} = s_{\bar{n}|i} \times (1+i)^{-n}$$
$$= \frac{(1+i)^n - 1}{i} \times (1+i)^{-n}$$
$$= \frac{1-(1+i)^{-n}}{i}$$

Example 5.3

How much money is needed now to provide RM500 at the end of each year (first payment 1 year from now) for 15 years if the money earns interest at 12% p.a?

Example 5.4

A student who borrowed some money to purchase a car was to repay the loan with monthly installments of RM150 for 3 years. Calculate the value of these repayments at the beginning of the loan if the interest rate was 9% convertible monthly.

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5.4 Annuity due

Annuity due is an annuity whose periodic payments are paid at the beginning of each payment period. The term of an annuity due starts at the time of the first payment and ends one payment period after the date of the last payment. This can be illustrated as figure below.

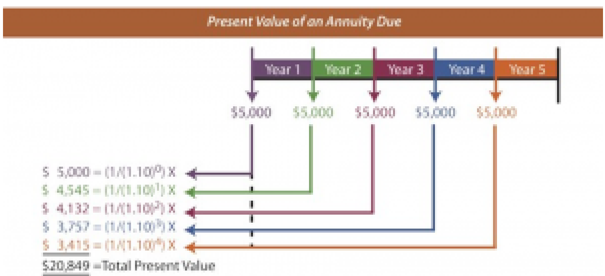

Figure 2: Illustration of an annuity due timeline

Annuity due also known as annuities payable in advance and normally arise in respect of insurance premiums, rents, etc.

We know that the future value of the payments at the end of the $(n - 1)$th period is R$s_{\bar{n}|i}$. We then accumulate it for 1 interest period, to obtain the future value of an annuity due, $s_{\ddot{n}|i}$ at the end of the term as;

Future value, $s_{\ddot{n}|i} = Rs_{\bar{n}|i}(1+i)$

While the present value of the payments one period before the first payment is R$a_{\bar{n}|i}$. To obtain the present value of an annuity due, R$a_{\ddot{n}|i}$ (i.e. on the date of the first payment), we accumulate R $a_{\bar{n}|i}$ for one interest period so that:

Present value, $a_{\ddot{n}|i}$ = R $a_{\bar{n}|i}(1+i)$

Example 5.5

Mrs. Mary deposits RM100 at the beginning of each year for 10 years in an account paying 12% p.a. How much is in her account at the end of 10 years?

Example 5.6

The monthly rent for a flat is RM520 payable at the beginning of each month. If money is worth i_{12} = 9%, what is the equivalent yearly rental payable in advance?

Example 5.7

A man saves RM15000 at the beginning of each year to accumulate a fund expansion. If the fund earns 15% p.a., what is the amount he will have at the end of 5 years?

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5.5 Review Exercises

Select TRUE or FALSE for each of the following statements.

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5.6 Highlights

- Annuity is a sequence of equal payments made at equal periods of time.
- When the payments are made at the end of the time period, the annuity is called an ordinary annuity.
- When the payments are paid at the beginning of each payment period, the annuity is called an annuity due.
- There is a significant difference between the present and future value for an ordinary annuity and an annuity due.

5.7 Key Terms

Annuity

- **Annuity due**
- **Future value**
- **Ordinary annuity**
- **Present value**

PART VI

CHAPTER 6: DEPRECIATION

Learning Outcomes

At the end of this chapter, you should able to:

- Define depreciation.
- Understand the various methods of calculating depreciation.

Assets purchased at a particular time may provide services over a finite future period, a notable exception for land. The account will allocate the cost of the asset to the accounting periods in which it generates revenues – a periodic expense item called depreciation. Among reasons for depreciation include time factor, decay, worn out, etc.

6.1 Depreciation in Accounting

Economic life of most assets is limited. After being used for a number of years, it will come to the end of their lives. As the time of retirement, assets may have a small trade-in or scrap value, or even may be worthless.

Computation of depreciation may involve compound interest and ordinary annuity. The computation of the periodic depreciation include allocating the cost of the property as an expense/ a cost of goods manufactured to the proper business operating periods. The amount of accumulated depreciation is known as accumulated depreciation or allowance for depreciation.

The difference between the original cost of an asset and its total amount in the accumulated depreciation is known as book value. Among reasons for depreciation include time factor, decay, worn out, etc.

In this chapter, we will use the following notation:

C = original cost of an asset

T = estimated trade-in value or scrap value

C – T = total depreciation charges or expenses

n = useful life of the asset estimated in years, service-hours, or product-units.

r = rate of depreciation expense per year, per service-hour, or per product-units.

6.1.1 Method of Averages

6.1.1.1 The Straight-Line Method

The simplest and most popular method of accounting for depreciation is the straight-line method. The depreciation charges are equal for each year. If R is the depreciation expense per year and n is the number of years, then R is the total depreciation charges divide by n, the number of years or can be written as:

$$R = \frac{C-T}{n}$$

Example 6.1

A machine was purchased for RM1,100 has an estimated useful life of 5 years and a trade-in value of RM120. Use the straight line method to find the depreciation charges for each year, and construct a depreciation schedule.

6.1.1.2 Service Hours Method

The service hours method relates depreciation to the estimated productive capacity of the asset in terms of its hours of useful service. The depreciation rate per service hour r is:

$$r = \frac{C-T}{n}$$

Where n is the number of service hours.

The depreciation charges for a given year is equal to r multiplies the number of service hours of that year.

Example 6.2

A machine was purchased for RM1,100 has an estimated useful life of 5 years and a trade-in value of RM120. Assume useful life of the machine is 20,000 service hours and actual number of service hours in each year is as follows.

Construct a depreciation schedule using the service hours method to find the depreciation charges.

1st year	5000 service hours
2nd year	4500 service hours
3rd year	4200 service hours
4th year	3400 service hours
5th year	2900 service hours

6.1.1.3 Product Units Method

The product units method defined depreciation to the estimated number of units produced by each asset during its useful life. The depreciation rate per unit product r is:

$$r = \frac{C-T}{n}$$

Where n is the number of product units. The depreciation charges for a given period equal to r multiplies the number of units produced in that period.

Example 6.3

A machine was purchased for RM1,100 has an estimated useful life of 5 years and a trade-in value of RM120. Assume useful life of machine is 70,000 product units and number of units produced each year is estimated as follows.

Use the product units method to find the depreciation charges for each year, and construct a depreciation schedule.

1st year	14000 units
2nd year	15000 units
3rd year	16500 units
4th year	17000 units
5th year	7500 units

6.1.2 Reducing Charge Method

6.1.2.1 Sum of the years-digit method

The total depreciation is fixed. It is the difference between the original cost and the trade-in or scrap value (C-T). The rate of depreciation is expressed in a changing fraction which becomes smaller each year. The numerator of the fraction is the number of remaining years of life of the asset. Whereas the denominator is the sum of digits that represent the years of life.

Example 6.4

A machine was purchased for RM1,100 has an estimated useful life of 5 years and a trade-in value of RM120. Use the sum of the years-digits method to find the depreciation charges for each year and construct the depreciation schedule.

6.1.2.2 Constant/ Fixed rate method on diminishing book value

The depreciation expense for earlier year is higher than the later years. The depreciation expense is equal to fixed/ constant annual rate multiply the diminishing book value of an asset. There are two ways of computation: i) the declining balance method, ii) the fixed rate method.

For the declining balance method, first we need to find the annual depreciation rate by dividing 100% by the number of years of useful life of the asset. Then, find the maximum annual depreciation rate by multiplying the annual depreciation rate obtained by 2.

For the fixed rate method, the book value at the end of the life of the asset is equal to the scrap or trade-in value. The rate of annual depreciation charges (r) can be written as:

$$r = 1 - \sqrt[n]{\frac{T}{C}}$$

6.1.3 Compound Interest Method

6.1.3.1 Annuity method

The annuity method resembles the method of amortizing a debt. The depreciation charges are equal and include not only a part of the cost of the asset but also the interest on the book value for each operating period. The step by step calculation are as follows:

Step 1: Find the present value of the total depreciation charges. The present value is, $P=T(1+i)^{-n}$

Step 2: The annual depreciation charges are required to be equal. Thus, we can assume that the present value of an annuity (A_n) with payments consisting of equal depreciation charges as:

$C-P = C- T(1+i)^{-n} = A_n = R\, a_{\bar{n}|i}$

Step 3: R represents the annual depreciation charges, and can be obtained as follows:

$$R = \frac{A_n}{a_{\bar{n}|i}} = \frac{C-T(1+i)^{-n}}{a_{\bar{n}|i}}$$

Example 6.5

A machine was purchased for RM1,100 has an estimated useful life of 5 years and a trade-in value of RM120. Use the annuity method to find the depreciation charges for each year and construct a depreciation schedule. Assume that the effective interest rate is 6%.

6.1.3.2 Sinking fund method

It is assumed that a sinking fund is established for the purpose of replacing an asset at the end of its useful life. The periodic depreciation charges are exactly the same as the periodic increases including the periodic deposit and interest) in the sinking fund. Hence, the depreciation charges are not equal. But the total of the depreciation charges is equal to the amount in the sinking fund (S_n) at the end of the useful life of the asset. The size of each deposit (R) made in the sinking fund is given by the annuity formula:

S_n = R$s_{\bar{n}|i}$ or R = $\frac{S_n}{s_{\bar{n}|i}} = \frac{C-T}{s_{\bar{n}|i}}$

6.1.4 Composite Rate Method

The composite rate method is used for computing the depreciation charges of a group of assets. The composite rate is obtained by dividing the total annual depreciation charges by the total cost of the group of assets. The annual depreciation charges of each asset are obtained by the straight line method. The composite rate may be used to compute depreciation charges during the later years if there is no significant changes in the values and useful lives of assets. The work of computing the depreciation expense of each item can be avoided. The composite rate of an asset can be calculated as:

The composite rate = $\frac{Total\ annual\ depreciation\ charges}{Total\ cost}$

The composite life of a group of assets is the average life of the group. In computing the composite life, the annual depreciation charges of the group of assets may be obtained by using: i) the composite rate, ii) the depreciation charges are assumed to be equal for each year.

The composite life = $\frac{Total\ depreciation\ charges}{Total\ annual\ depreciation\ charges}$

Example 6.6

Find the composite rate and composite life of the group of assets in the table below:

Asset	Original cost	Scrap value	Estimated life
A	RM10000	9000	10 years
B	5000	4800	12 years
C	4500	4225	5 years

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6.1.4.1 The sinking fund method

If the depreciation charges are not equal for each year, it is not categorised as a fixed composite rate. The annual deposits in the fund for each asset are equal during the life of the asset. The composite life is the time necessary for the total annual deposits at the given interest rate to accumulate to the total depreciation charges of the group of assets.

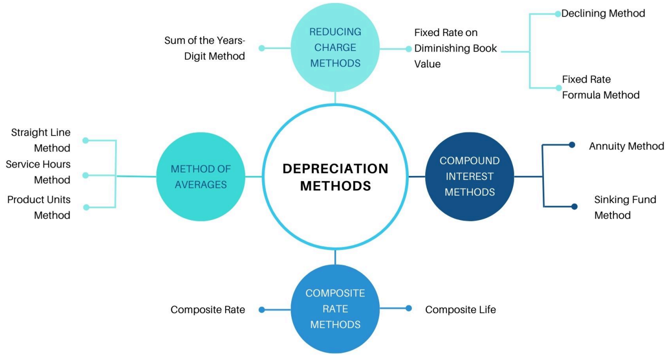
Example 6.7

Find the composite life of the group of assets in the table below using the sinking fund method. Assume that the effective interest rate is 6%.

Asset	Original cost	Scrap value	Estimated life
A	RM10000	9000	10 years
B	5000	4800	12 years
C	4500	4225	5 years

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://openbook.ums.edu.my/financialmathematicsineconomics/?p=110#h5p-34

6.2 Flowchart on Depreciation Method



6.3 Review Exercises

Select TRUE or FALSE for each of the following statements.

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6.4 Highlights

- Depreciation allows businesses to spread the cost of tangible assets over a period of years for accounting and tax purposes.
- Among reasons for depreciation include time factor, decay, worn out, etc.
- Four methods in calculating depreciation expenses are the method of averages, reducing charge method, compound interest method, and composite rate method.

6.5 Key Terms

Accumulated depreciation

Book value

Depreciation

Appendix: Glossary of Formulas and Definitions

No.	**Formula**	**Definition**
1.	$I = P \times r \times t$	Simple interest
2.	$S = P + I = P + (Prt) = P(1 + rt)$	Future value of simple interest
3.	$P = \frac{S}{1 + rt}$	Present value of simple interest
4.	$P = S(1 - dt)$	Present value of simple discount
5.	$S = \frac{P}{1 - dt}$	Future value of simple discount
6.	$S = P(1 + i)^n$	Compound interest
7.	$S = P(1 - \frac{d^m}{m})^{-n}$	Compound discount at discount rate

8.	$r_E = (1 + \frac{r}{m})^m - 1$	Effective rate
9.	D = Sdt	Bank discount
10.	List price x Discount rate	Trade discount
11.	$s_{\bar{n}\|i} = \frac{(1+i)^n - 1}{i}$	Future value of an ordinary annuity
12.	$a_{\bar{n}\|i} = \frac{1-(1+i)^{-n}}{i}$	Present value of an ordinary annuity
13.	$s_{\ddot{n}\|i} = s_{\bar{n}\|i}(1 + i)$	Future value of an annuity due
14.	$a_{\ddot{n}\|i} = a_{\bar{n}\|i}(1 + i)$	Present value of an annuity due

Glossary

Accumulated depreciation

The amount of accumulated depreciation.

Annuity

A sequence of equal payments made at equal periods of time.

Annuity due

An annuity whose periodic payments are paid at the beginning of each payment period.

Bank discount

A bank charge that is made for payment of a note at some point prior to maturation.

Book value

The difference between the original cost of an asset and its total amount in the accumulated depreciation.

Cash discount

A discount to the selling price to induce early payment.

Compound interest

When the interest of each period is added to the principal in computing the interest for the next period.

Depreciation

An accounting method used to allocate the cost of a tangible or physical asset over its useful life.

Discount rate

An interest rate used to remove interest from a future value.

Economic gains

Economic gain or economic profit refers to the difference between the revenue received from the sale of an output and the costs of all inputs used, as well as any opportunity costs.

Future value

The total value of the set of payments at the end of the term

Inflation

A rise in prices, which can be translated as the decline of puchasing power over time.

Interest

The amount of interest that is paid or earned.

Interest rate

The amount a lender charges a borrower and is a percentage of the principal-the amount loaned.

Negotiable instrument

A document guaranteeing the payment of a specific amount of money, either on demand or at a set time, with the payer named on the document.

Opportunity cost

The potential benefits that an individual, investor, or business misses out on when choosing one alternative over another.

Ordinary annuity

An annuity whose periodic payments are made at the end of the time period.

Present value

The amount of money at the beginning of a time period in a transaction.

Principal

The original amount of money that is borrowed or invested in a financial transaction. Also called as the present value.

Saving

The amount of money left over after spending and other obligations are deducted from earnings.

Simple interest

A system for calculating interest that primarily applies to, in general, short-term financial transactions with a time frame of less than one year.

Time period

The length of the financial transaction for which interest is charged or earned. It may also called the term.

Time value of money

A financial principle that states the value of a dollar today is worth more than the value of a dollar in the future.

Trade discount

A discount that is cut from the retail or published price of an item.

References

Erickson, K.H. 2014. Financial Economics: A Simple Introduction. CreateSpace Independent Publishing Platform.

Knox D.M., Zima P., Brown R.L. 2011. Mathematics of Finance. New York: McGraw-Hill.

Mishkin, F.S. 2018. The economics of money, banking, and financial markets. Pearson.

Shao S.P. & Shao L.P. 1998. Mathematics for Management and Finance. South-Western College Publishing.

Stampfli J. & Goodman, V. 2008. The Mathematics of Finance: Modeling and Hedging. Brooks/Cole Thompson Learning.

Stephen G.K. 2009. The Theory of Interest. 3rd Ed. Singapore: McGraw-Hill.

Wilders, R.J. 2020. Financial Mathematics for Actuarial Science. The Theory of Interest. 1st Ed. CRC Press.